AP CALCULUS AB and BC

Summer Assignment for students entering AP Calculus

There are 150 questions that need to be answered completely. Show all work. This will be collected the first Monday of school and will count as two quiz grades for the first quarter with an in class test the following week. If you need help with these questions, please email me over the summer with specifics and I will do the best I can to get back to you in a timely manner.

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Please do not simply copy each other's work... Sometimes this leads to wrong answers spreading like a wild fire. Also do not complete in June... you are better off carving out some time in August, this way you will be more familiar with what you need for September.

I am looking forward to an exciting year of AP Calculus!

Mr. Maggio

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ___________ 7. $g(-3) =$ ___________ 8. $f(t+1) =$ ___________

9. $f[g(-2)] =$ ___________ 10. $g[f(m+2)] =$ ___________ 11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ ___________ 13. $f\left(\dfrac{2\pi}{3}\right) =$ ___________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

14. $h[f(-2)] =$ ______ 15. $f[g(x-1)] =$ ______ 16. $g[h(x^3)] =$ ______

Find $\frac{f(x+h)-f(x)}{h}$ for the given function *f*.

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int.}\ (\textit{Let } y=0)$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1 \textit{ or } x=3$

$x-\text{intercepts } (-1,0) \textit{ and } (3,0)$

$y-\text{int.}\ (\textit{Let } x=0)$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept } (0,-3)$

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Use substitution or elimination method to solve the system of equations.
Example:

$x^2 + y - 16x + 39 = 0$

$x^2 - y^2 - 9 = 0$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ and } x = 5$

Plug x=3 *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection (5,4), (5,–4) *and* (3,0)

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

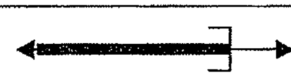
23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

25. $x^2y + y^2 = 2x$
$x^2 + 2y = 0$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1,7)$	
		 8

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \dfrac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right)+9 \qquad g(f(x)) = \frac{(4x+9)-9}{4}$$

$$= x-9+9 \qquad = \frac{4x+9-9}{4}$$

$$= x \qquad = \frac{4x}{4}$$

$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and* g are inverses of each other.

36. $f(x) = \dfrac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2,\ x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Derivatives and Critical Points

Use the power rule to find the derivative for each of the following functions.

Power Rule: *Given* $f(x) = ax^n, \quad f'(x) = n \cdot ax^{n-1}$

Example:	$f(x) = 3x^2 - 5x + 7$ $f'(x) = 6x - 5$

46. $f(x) = 5x^4 - 3x^2 + 2$

47. $f(x) = 6x^3 - 5x^2 + 4x - 2$

48. $f(x) = 2x + 4$

49. $f(x) = 7$

50. $f(x) = x^5 - 2x^3 - 5x + 4$

Find the slope of the line tangent to each of the following functions at the given point. Recall that the derivative of a function is a formula for the slope of the tangent line at any point along the function. To find the slope at a given point, use the power rule to determine the derivative and substitute the x-coordinate into the derivative.

Example:	$f(x) = 4x^3 - 2x + 1$ $f'(x) = 12x^2 - x$ at the point $(1, 3)$, the slope of the tangent line is $f'(1) = 12(1)^2 - 1 = 11$

51. $f(x) = x^4 - 2x^3 + 5x^2 - 8$ at the point (-2, 44)

52. $f(x) = -x^2 - x + 2$ at the point (0.5, 1.25)

Find the equation of the line in slope intercept form, tangent to the graph of each of the following functions at the given point. Use the derivative as in the previous example to find the slope. Use the slope and the given point to write the equation of the line tangent to the graph.

Example:	$f(x)=x^2-2x+7$ at the point $(2,7)$		
	$f'(x)=2x-2$		
	$f'(2)=2(2)-2=2$		
	$y-y_1=m(x-x_1)$	or	$y=mx+b$
	$y-7=2(x-2)$		$7=2(2)+b$
	$y=2x+3$		$y=2x+3$

53. $f(x)=-2x^2+3x+10$ at the point (1, 11)

54. $f(x)=3x^3-2x^2+4x-2$ at the point (-2, -42)

Find the critical numbers for each of the following functions. Critical numbers are the x-values for which the derivative is undefined or equal to zero. Find the derivative and set it equal to zero.

Example:	$f(x)=\frac{2}{3}x^3+\frac{5}{2}x^2-7x$
	$f'(x)=2x^2+5x-7$
	$0=(x-1)(2x+7)$
	Critical numbers: $x=1$ or $x=-\frac{7}{2}$

55. $f(x)=3x^3-9x+5$

56. $f(x)=x^2+2x-15$

Determine the maximum or minimum value(s) of the function. First, find the critical numbers, then use a sign chart to test whether a maximum value, a minimum value or neither exists at the critical number.

Example:

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

critical numbers: $x = 0, x = 2$

Sign chart: $f'(-1) > 0$ $\quad f'(1) < 0$ $\quad f'(3) > 0$

0 $\quad$ 2

The sign of the derivative changes from positive to negative at x = 0, so we have a **maximum** at the point (0, 0). The sign of the derivative changes from negative to positive at x = 2, so we have a **minimum** at the point (2, -4).

57. $f(x) = x^2 - 2x$

58. $f(x) = x^3 - 12x$

Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.

Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

59. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

60. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

61. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

62. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

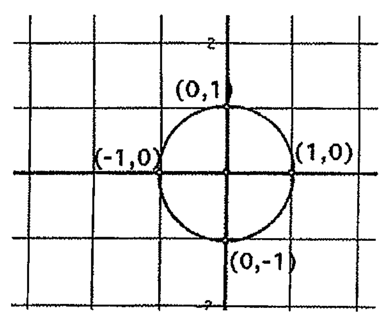
a. $\frac{2}{3}\pi$ b. 225°

c. $-\frac{\pi}{4}$ d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

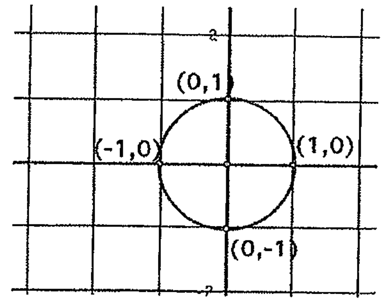
Example: $\sin 90^\circ = 1$ $\cos\frac{\pi}{2} = 0$



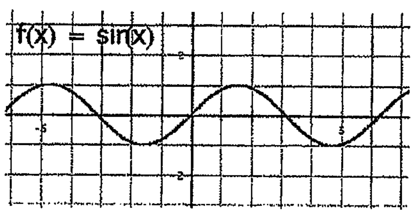
63. $a.)\ \sin 180^\circ$ $b.)\ \cos 270^\circ$

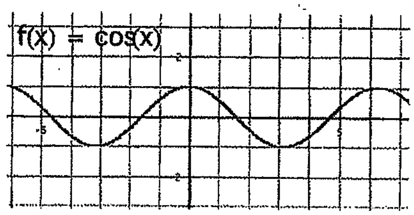
$c.)\ \sin(-90^\circ)$ $d.)\ \sin \pi$

$e.)\ \cos 360^\circ$ $f.)\ \cos(-\pi)$



Graphing Trig Functions





y = sin x and y = cos x have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

64. $f(x) = 5\sin x$

65. $f(x) = \sin 2x$

66. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

67. $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

68. $\sin x = -\frac{1}{2}$

69. $2\cos x = \sqrt{3}$

70. $\cos 2x = \frac{1}{\sqrt{2}}$

71. $\sin^2 x = \frac{1}{2}$

72. $\sin 2x = -\frac{\sqrt{3}}{2}$

73. $2\cos^2 x - 1 - \cos x = 0$

74. $4\cos^2 x - 3 = 0$

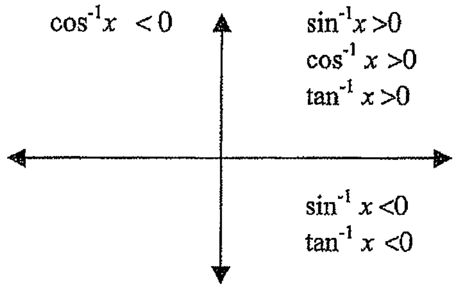
75. $\sin^2 x + \cos 2x - \cos x = 0$

<u>Inverse Trigonometric Functions:</u>

Recall: Inverse Trig Functions can be written in one of ways:

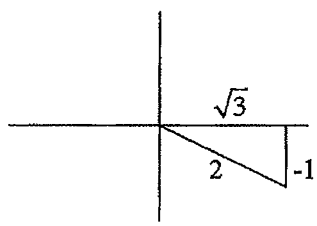
$\arcsin(x)$ $\sin^{-1}(x)$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains:



Example:
Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: $y = -\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

76. $y = \arcsin\frac{-\sqrt{3}}{2}$

77. $y = \arccos(-1)$

78. $y = \arctan(-1)$

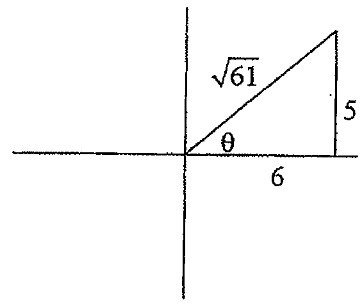
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

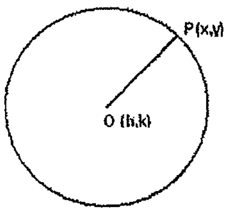
79. $\tan\left(\arccos\frac{2}{3}\right)$

80. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

81. $\sin\left(\arctan\frac{12}{5}\right)$

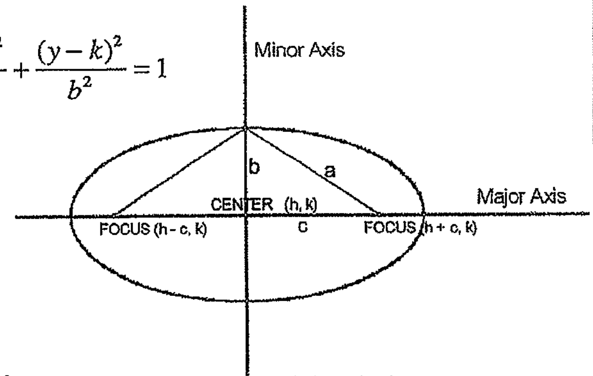
82. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

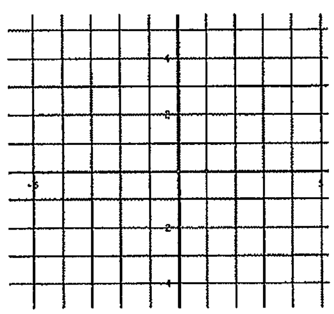


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

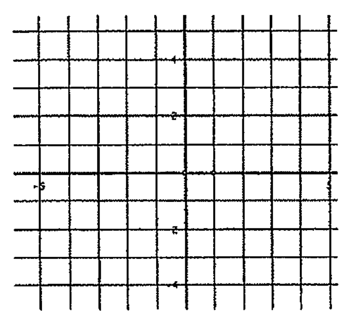
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the y-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

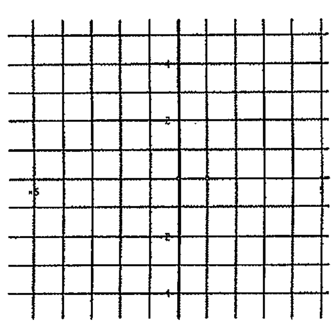
83. $x^2 + y^2 = 16$



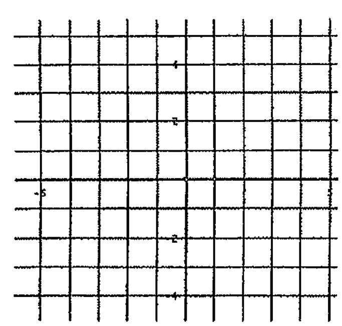
84. $x^2 + y^2 = 5$



85. $\frac{x^2}{1} + \frac{y^2}{9} = 1$



86. $\frac{x^2}{16} + \frac{y^2}{4} = 1$



Limits

Finding limits numerically.

Complete the table and use the result to estimate the limit.

87. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

88. $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically.

Find each limit graphically. Use your calculator to assist in graphing.

89. $\lim_{x \to 0} \cos x$

90. $\lim_{x \to 5} \frac{2}{x-5}$

91. $\lim_{x \to 1} f(x)$

$$f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Evaluating Limits Analytically

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

92. $\lim_{x \to 2} (4x^2+3)$

93. $\lim_{x \to 1} \frac{x^2+x+2}{x+1}$

94. $\lim_{x \to 0} \sqrt{x^2+4}$

95. $\lim_{x \to \pi} \cos x$

96. $\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)$ HINT: Factor and simplify.

97. $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

98. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ HINT: Rationalize the numerator.

99. $\lim_{x\to 3}\frac{3-x}{x^2-9}$

100. $\lim_{h\to 0}\frac{2(x+h)-2x}{h}$

One-Sided Limits

Find the limit if it exists. First, try to solve for the overall limit. If an overall limit exists, then the one-sided limit will be the same as the overall limit. If not, use the graph and/or a table of values to evaluate one-sided limits.

101. $\lim_{x\to 5^+}\frac{x-5}{x^2-25}$

102. $\lim_{x\to -3^-}\frac{x}{\sqrt{x^2-9}}$

103. $\lim_{x\to 10^+}\frac{|x-10|}{x-10}$

104. $\lim_{x\to 5^-}\left(-\frac{3}{x+5}\right)$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

105. $f(x)=\dfrac{1}{x^2}$

106. $f(x)=\dfrac{x^2}{x^2-4}$

107. $f(x)=\dfrac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y=0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

108. $f(x)=\dfrac{x^2-2x+1}{x^3+x-7}$

109. $f(x)=\dfrac{5x^3-2x^2+8}{4x-3x^3+5}$

110. $f(x)=\dfrac{4x^5}{x^2-7}$

Limits to Infinity

A rational function does not have a limit if it goes to $\pm\infty$, however, you can state the direction the limit is headed if both the left and right hand side go in the same direction.

Determine each limit if it exists. If the limit approaches ∞ *or* $-\infty$, please state which one the limit approaches.

111. $\lim\limits_{x\to-1^+}\dfrac{1}{x+1}=$

112. $\lim\limits_{x\to1^+}\dfrac{2+x}{1-x}=$

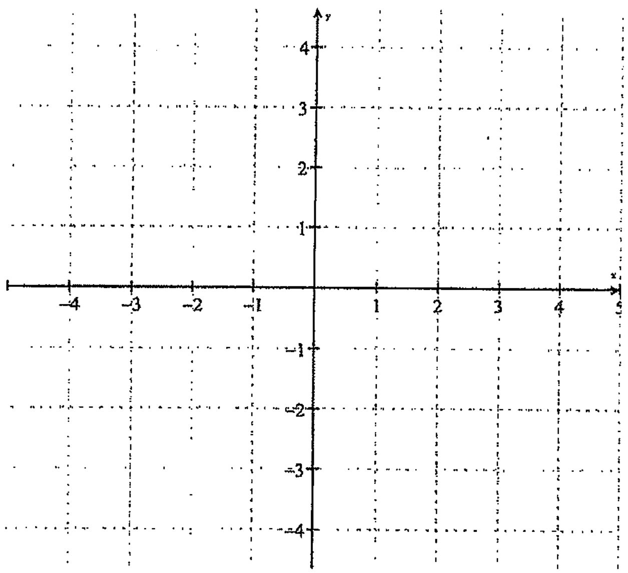
113. $\lim\limits_{x\to0}\dfrac{2}{\sin x}=$

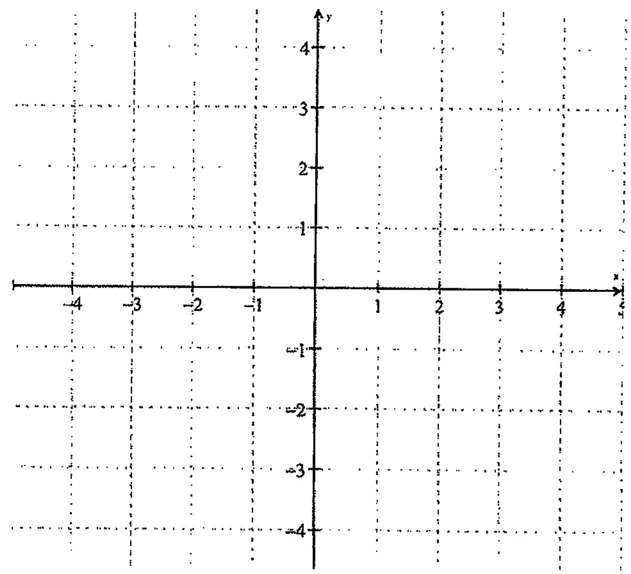
Piecewise Functions

Sketch the graph of each of the following piecewise-defined functions on the axes provided.

114. $f(x)=\begin{cases}\sqrt{-2-x} & \text{if } x<-2\\ x^2-4 & \text{if } -2\le x<1\\ \ln x & \text{if } x>1\end{cases}$

115. $g(x)=\begin{cases}|x+1|+2 & \text{if } x\le 0\\ \sqrt{9-x^2} & \text{if } 0<x<3\\ \sin(\pi x) & \text{if } x\ge 3\end{cases}$





Based on the functions *f* and *g* defined in questions 114 and 115, determine the following limits or state that they do not exist.

116. $\lim_{x\to 0} f(x)$

117. $\lim_{x\to 1^+} f(x)$

118. $\lim_{x\to 1} f(x)$

119. $\lim_{x\to -2} f(x)$

120. $\lim_{x\to 0} g(x)$

121. $\lim_{x\to 3} g(x)$

122. $\lim_{x\to -1}[3f(x)-5g(x)]$

123. $\lim_{x\to 4}[kg(x)-mf(x)]$ (write your answer in terms of *k* and *m*, which are unknown constants)

124. Determine all values of *a* that would make the piecewise function *h* below continuous at $x=2$.

$$h(x)=\begin{cases}\log_2(x^2+4) & \text{if } x<2\\ ax^2+ax-4 & \text{if } x\ge 2\end{cases}$$

Evaluating Logarithmic Expressions and Properties of Logarithms

1. **Logarithmic/Exponential Equivalency:** $\log_b x = y \leftrightarrow b^y = x$
2. **Exponent Property of Logarithms:** $\log_b(x^y) = y\log_b x$
3. **Addition Property of Logarithms:** $\log_b x + \log_b y = \log_b xy$
4. **Subtraction Property of Logarithms:** $\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

Remember: $\log x$ means $\log_{10} x$ **and** $\ln x$ means $\log_e x$

Convert each exponential statement into a logarithmic statement.

125. $2^y = 16$

126. $10^a = b$

127. $e^{5x} = y$

Convert each logarithmic statement into an exponential statement.

128. $\log_3(81) = 4$

129. $\log 1000 = 3$

130. $\ln x = y$

Without a calculator, find the value of each expression below.

131. $\log_2(32)$

132. $\log\left(\frac{1}{100}\right)$

133. $\ln(e^6)$

134. $\ln 1$

Write each expression using only one logarithm. Simplify if possible.

135. $\log a^2 + \frac{1}{2}\log b - 3\log c$

136. $\log_3(x^2 + x - 6) - \log_3(x+3)$

Expand each logarithmic expression into the sum and difference of logarithmic expressions. Simplify if possible.

137. $\log\left(\frac{10x^3}{\sqrt{y}}\right)$

138. $\ln\left(\frac{xyz}{e^2}\right)$

Logarithmic and Exponential Expressions and Equations

Solving Exponential Equations:

Example 1: Solve for x: $9^{3x} = 27^{x+2}$

You must make the bases the same number. In this case, both 9 and 27 are powers of 3.

$$\left(3^2\right)^{3x} = \left(3^3\right)^{x+2}$$
$$6x = 3x + 6$$
$$3x = 6$$
$$x = 2$$

Example 2: Solve for x: $e^{3x} - 6 = 5$

When the bases cannot be made the same, you must use logarithms after the base to a power is isolated.

$$e^{3x} - 6 = 5$$
$$e^{3x} = 11$$
$$3x \ln e = \ln 11 \qquad \ln e = 1 \text{ since } e^1 = e$$
$$3x = \ln 11$$
$$x = \frac{\ln 11}{3}$$

Solve each equation for x:

139. $4^x = 8^{x-2}$

140. $7^{4x} + 6 = 17$

141. $e^{6x} + 4 = 20$

142. $e^{2x} - 6e^x + 5 = 0$ [Hint: Let $y = e^x$ and note that $e^{2x} = \left(e^x\right)^2$]

Logarithmic Equations

Example 1: Solve for x: $\ln(x-1)=2$

Note that $\ln(x-1)=\log_e(x-1)$ and rewrite as an exponential statement:

$\log_e(x-1)=2 \leftrightarrow e^2=x-1$ so $x=e^2+1$

Example 2: Solve for x: $\log_2(x+2)+\log_2(x+6)=5$

When there is more than one log on one side, condense to make one log.

$\log_2\left((x+2)(x+6)\right)=5$

$\log_2\left(x^2+8x+12\right)=5$

$2^5=x^2+8x+12$

$32=x^2+8x+12$

$0=x^2+8x-20$

$0=(x+10)(x-2)$

$x=-10,2$ Reject -10 since it makes quantities inside the logs negative, so $x=2$ is the only solution.

Questions 143-146: Solve for x in each case.

143. $\ln x=5$

144. $\log x=1.35$

145. $3\ln(x-5)-2=7$ [Hint: Isolate ln first]

146. $\log_5(x+1)+\log_5(2x-3)=2$

147. Solve for y: $\ln(y-1)=x^3+C$ [Your answer will be in terms of x and C]

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148. If $F(x)=\frac{x^3}{3}+\frac{x^2}{2}-3x+C$ and $F(2)=3$, find the value of the constant C.

149. Write an equivalent expression using no radicals and using no denominators: $3\sqrt{x}-4\sqrt[3]{x^2}-\frac{2}{\sqrt{x}}$

150. Solve for z in terms of x and y: $2xy+x^2z=x^2y-3yz$

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$ $= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0